Sujet de Thèse

- **Titre :** Full dispersion models in oceanography
- **Unité de recherche :** IRMAR, UMR-6625
- **Thème :** Analyse EDP
- **Mots clefs :** Nonlinear dispersive equations, asymptotic models, gravity waves
- **Les noms, prénoms et courriel du directeur de thèse**

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Description

The justification of long waves models for the propagation of surface gravity waves, such as the celebrated Korteweg-de Vries (KdV) equation

$$\partial_t u + \sqrt{gd}\left(\partial_x u + \frac{3}{2d}u\partial_x u + \frac{d^2}{6}\partial_x^3 u\right) = 0,$$

is by now well-understood. By construction, these models fit the behavior of the “exact” system of equations (the water-waves system) only for the low-frequency component of the wave, but typically provide quaint predictions for the large-frequency component (looking for instance at the relation dispersion). As a remedy, Whitham [7] proposed the following *ad-hoc* modification of the KdV equation :

$$\partial_t u + \sqrt{gd}\left(\sqrt{\frac{\tanh(dD_x)}{dD_x}}u + \frac{3}{2d}\zeta\partial_x u\right) = 0,$$

This model has the “full dispersion” property in the sense that its linearization about trivial states fits exactly with the one of the water-waves system (once a direction of propagation has been selected). Although the Whitham model produces qualitatively better predictions than the KdV model [6] and has attracted a fair amount of attention lately [3], it is for now justified only formally [5].

The first aim of the thesis would be to follow the strategy provided in [4] for long waves models in order to offer a rigorous justification of full dispersion models existing in the literature, or produce relevant new ones. This would allow to quantify the precision and regime of validity of such models — which we expect to contain both shallow-water long waves, and small amplitude high-frequency waves.

Once a rigorous ground has been given to such full-dispersion models, one should be able to exploit them in order to provide relevant information on phenomenons which arise in the afore-described regime. One can think at

- the influence of a “rough” bottom topography on the propagation of long waves [2] ;
- the interaction between (typically long and large amplitude) internal waves and (typically high-frequency and small amplitude) surface waves in a density-stratified fluid [1].

References

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